

# Extended propagation-inside-layer expansion method combined with the forward-backward method to study the scattering from an object above a rough surface

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In this Letter, a fast and rigorous numerical method, based on the method of moments, is developed to calculate the scattering from an object above a rough surface for three-dimensional problems (3D). G. Kubické has recently developed the extended propagation-inside-layer expansion (E-PILE) method to calculate the scattering from an object above a rough surface for two-dimensional problems. This method allows us to calculate separately and exactly the interactions between the object and the rough surface. The purpose of this paper is to extend the E-PILE method to a 3D problem. In addition, to invert a matrix of large size, the forward-backward (FB) method is applied to calculate the local interactions on the rough surface. © 2012 Optical Society of America

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The study of electromagnetic scattering from an object above a rough surface has attracted considerable interest in the fields of optics and remote sensing. Some methods have been developed for the efficient analysis of composite scattering for a two-dimensional problem (2D) [1–4]. However it is important for practical applications to study the case of a three-dimensional (3D) problem [5–9]. For a large 3D problem, it is well known that the method of moments (MoM) is limited by the memory requirement and computation time, so some simplifying assumptions must be made. Déchamps *et al.* [1,2] developed a fast numerical method, propagation-inside-layer expansion (PILE), to study the scattering from a rough layer. In that method, only the upper surface is illuminated. Recently, G. Kubické *et al.* [3,4] extended the PILE method to the case of an object above a one-dimensional (1D) rough surface, in which the two scatterers are illuminated. The purpose of this paper is to extend the E-PILE method to the more general 3D case of an object above a large rough surface, and to accelerate it by using the forward-backward (FB) method of Holliday *et al.* [10] for the calculation of the local interactions on the rough surface.

Let us consider an incident electromagnetic wave that illuminates the system, composed of two perfect electric conductor (PEC) scatterers (object + rough surface). The unitary (in amplitude) incident electromagnetic fields are written as

$$\mathbf{E}^i(\mathbf{R}) = \hat{\mathbf{e}}_i e^{ik_0 \mathbf{k}_i \cdot \mathbf{R}}, \quad \mathbf{H}^i(\mathbf{R}) = \frac{1}{\eta_0} \hat{\mathbf{k}}_i \times \mathbf{E}^i(\mathbf{R}), \quad (1)$$

where  $\mathbf{k}_i = k_0 \hat{\mathbf{k}}_i$ , and  $k_0$  and  $\eta_0$  are, respectively, the wave number and impedance of free space. Note that the harmonic time convention  $e^{-i\omega t}$  is assumed and suppressed throughout this paper.

By using the boundary conditions on scatterer 1, object  $S_1$ , the first coupling integral equation between  $S_1$  and  $S_2 \forall \mathbf{R}' \in S_1$  is obtained from

$$\begin{aligned} \hat{\mathbf{n}}_1 \times \mathbf{H}^i(\mathbf{R}') + \underbrace{\hat{\mathbf{n}}_1 \times \int_{S_1} \mathbf{J}_1(\mathbf{R}_1) \times \nabla_{\mathbf{R}_1} G(\mathbf{R}_1, \mathbf{R}') dS}_{\text{local interactions}} \\ + \underbrace{\hat{\mathbf{n}}_1 \times \int_{S_2} \mathbf{J}_2(\mathbf{R}_2) \times \nabla_{\mathbf{R}_2} G(\mathbf{R}_2, \mathbf{R}') dS}_{\text{coupling interactions}} \\ = \frac{1}{2} \mathbf{J}_1(\mathbf{R}'), \end{aligned} \quad (2)$$

where  $\int_S dS$  is the principal value integral,  $G(\mathbf{R}_1, \mathbf{R}') = \exp(ik_0 r)/4\pi r$  is the free-space Green function with  $r$  the distance between the two points  $\mathbf{R}_1$  and  $\mathbf{R}'$ , and  $\mathbf{J}_1$ ,  $\mathbf{J}_2$  are the electric currents on the two scatterers  $S_1$  and  $S_2$ , respectively.

Using the boundary conditions on scatterer 2, rough surface  $S_2$ , the second coupling integral equation  $\forall \mathbf{R}' \in S_2$  is obtained from

$$\begin{aligned} \hat{\mathbf{n}}_2 \times \mathbf{H}^i(\mathbf{R}') + \underbrace{\hat{\mathbf{n}}_2 \times \int_{S_2} \mathbf{J}_2(\mathbf{R}_2) \times \nabla_{\mathbf{R}_2} G(\mathbf{R}_2, \mathbf{R}') dS}_{\text{local interactions}} \\ + \underbrace{\hat{\mathbf{n}}_2 \times \int_{S_1} \mathbf{J}_1(\mathbf{R}_1) \times \nabla_{\mathbf{R}_1} G(\mathbf{R}_1, \mathbf{R}') dS}_{\text{coupling interactions}} \\ = \frac{1}{2} \mathbf{J}_2(\mathbf{R}'), \end{aligned} \quad (3)$$

where  $G(\mathbf{R}_2, \mathbf{R}') = \exp(ik_0 r)/(4\pi r)$  is the free-space Green function with  $r$  the distance between the two points  $\mathbf{R}_2$  and  $\mathbf{R}'$ .

The use of the MoM with point matching and pulse basis functions leads to the following linear system:

$$\bar{\mathbf{Z}}\mathbf{X} = \mathbf{b}, \quad (4)$$

where  $\bar{\mathbf{Z}}$  is the impedance matrix of the total scene ( $S_1 \cup S_2$ ) of size  $(N_1 + N_2) \times (N_1 + N_2)$ . The unknown vector  $\mathbf{X}$  of length  $(N_1 + N_2)$  is equal to  $\mathbf{X}^T = [\mathbf{X}_1^T \ \mathbf{X}_2^T]$ , where  $T$  stands for transpose operator.  $\mathbf{X}_j$  ( $j = 1, 2$ ) of length  $N_j$  contains the unknown currents on each scatterer

$$\mathbf{X}_j^T = \left[ \underbrace{\mathbf{J}(\mathbf{R}_j^1) \dots \mathbf{J}(\mathbf{R}_j^{N_j})}_{\text{Scatterer } j} \right], \quad (5)$$

in which  $\mathbf{J}(\mathbf{R}_j) = J_{xj}\hat{\mathbf{x}} + J_{yj}\hat{\mathbf{y}} + J_{zj}\hat{\mathbf{z}}$ . The source term  $\mathbf{b}$  is defined as  $\mathbf{b}^T = [\mathbf{b}_1^T \ \mathbf{b}_2^T]$ , where

$$\mathbf{b}_j^T = \left[ \underbrace{\hat{\mathbf{n}}_j \wedge \mathbf{H}^i(\mathbf{R}_j^1) \dots \hat{\mathbf{n}}_j \wedge \mathbf{H}^i(\mathbf{R}_j^{N_j})}_{\text{Scatterer } j} \right]. \quad (6)$$

To solve the linear system, the matrix  $\bar{\mathbf{Z}}$  is expressed from submatrices as

$$\bar{\mathbf{Z}} = \begin{bmatrix} \bar{\mathbf{Z}}_1 & \bar{\mathbf{Z}}_{21} \\ \bar{\mathbf{Z}}_{12} & \bar{\mathbf{Z}}_2 \end{bmatrix}. \quad (7)$$

$\bar{\mathbf{Z}}_1$  corresponds to the impedance matrix of the first scatterer as if it is assumed to be alone (in free space), and  $\bar{\mathbf{Z}}_2$  corresponds to the impedance matrix of the second scatterer as if it is assumed to be alone (in free space).  $\bar{\mathbf{Z}}_{21}$  and  $\bar{\mathbf{Z}}_{12}$  can be interpreted as coupling matrices for the interaction between  $S_1$  and  $S_2$ . Inverting by block the impedance matrix and using the way in [1], we can show after some manipulations that the unknown current on the upper scatterer  $\mathbf{X}_1$  is expressed as

$$\begin{aligned} \mathbf{X}_1 &= \left[ \sum_{p=0}^{p=P_{\text{E-PILE}}} \bar{\mathbf{M}}_{c,1}^p \right] \bar{\mathbf{Z}}_1^{-1} (\mathbf{b}_1 - \bar{\mathbf{Z}}_{21} \bar{\mathbf{Z}}_2^{-1} \mathbf{b}_2) \\ &= \sum_{p=0}^{p=P_{\text{E-PILE}}} \bar{\mathbf{X}}_1^{(p)}, \end{aligned} \quad (8)$$

in which

$$\begin{cases} \mathbf{X}_1^{(0)} = \bar{\mathbf{Z}}_1^{-1} (\mathbf{b}_1 - \bar{\mathbf{Z}}_{21} \bar{\mathbf{Z}}_2^{-1} \mathbf{b}_2) & \text{for } p = 0 \\ \mathbf{X}_1^{(p)} = \bar{\mathbf{M}}_{c,1} \bar{\mathbf{X}}_1^{(p-1)} & \text{for } p > 0 \end{cases}. \quad (9)$$

$\bar{\mathbf{M}}_{c,1}$  is a characteristic matrix expressed as

$$\bar{\mathbf{M}}_{c,1} = \bar{\mathbf{Z}}_1^{-1} \bar{\mathbf{Z}}_{21} \bar{\mathbf{Z}}_2^{-1} \bar{\mathbf{Z}}_{12}. \quad (10)$$

The unknown vector  $\mathbf{X}_2$  is obtained by substituting in Eqs (8), (9), and (10) subscripts 1, 2, 12, and 21 for subscripts 2, 1, 21, and 12, respectively.  $\bar{\mathbf{M}}_{c,1}$  has a clear physical interpretation: in the zeroth order terms,  $\bar{\mathbf{Z}}_1^{-1}$  accounts for the local interactions on the upper scatterer  $S_1$ , so  $\mathbf{X}_1^{(0)}$  corresponds to the contribution of the scattering on the upper scatterer, when it is illuminated by the direct incident field ( $\mathbf{b}_1$ ) and the direct scattered field by the lower scatterer  $S_2$  ( $-\bar{\mathbf{Z}}_{21} \bar{\mathbf{Z}}_2^{-1} \mathbf{b}_2$ ). In the first order

term,  $\mathbf{X}_1^{(1)} = \bar{\mathbf{M}}_{c,1} \mathbf{X}_1^{(0)}$ ,  $\bar{\mathbf{Z}}_{12}$  propagates the resulting upper field information,  $\mathbf{X}_1^{(0)}$ , toward the lower scatterer,  $\bar{\mathbf{Z}}_2^{-1}$  accounts for the local interactions on this scatterer, and  $\bar{\mathbf{Z}}_{21}$  repropagates the resulting contribution toward the upper scatterer; finally,  $\bar{\mathbf{Z}}_1^{-1}$  updates the field values on the upper scatterer. Thus, the characteristic matrix  $\bar{\mathbf{M}}_{c,1}$  realizes a back and forth between the upper scatterer and the lower one. The order  $P_{\text{E-PILE}}$  of E-PILE corresponds to the  $P_{\text{E-PILE}}$  back and forth between the upper and the lower scatterer. In the same manner,  $\bar{\mathbf{M}}_{c,2}$  realizes a back and forth between the lower scatterer and the upper one.

One of the advantages of the E-PILE method is that it can be combined with algorithms valid for a single scatterer, so the FB method is applied to speed up the calculation of  $\bar{\mathbf{Z}}_2^{-1} \mathbf{b}_2$  in Eq. (9), in order to reduce the complexity to  $\mathcal{O}(N_2^2)$  instead of  $\mathcal{O}(N_2^3)$  from a direct lower upper (LU) inversion. We want to solve  $\bar{\mathbf{Z}}_2 \mathbf{U} = \mathbf{b}_2 \Leftrightarrow \bar{\mathbf{Z}}_2^{-1} \mathbf{b}_2 = \mathbf{U}$ . The FB algorithm decomposes  $\bar{\mathbf{Z}}_2 \mathbf{U} = \mathbf{b}_2$  as

$$\bar{\mathbf{Z}}_d \mathbf{U}_f = \mathbf{b}_2 - \bar{\mathbf{Z}}_f (\mathbf{U}_b + \mathbf{U}_f), \quad (11)$$

$$\bar{\mathbf{Z}}_d \mathbf{U}_b = -\bar{\mathbf{Z}}_b (\mathbf{U}_b + \mathbf{U}_f). \quad (12)$$

$\bar{\mathbf{Z}}_d$  is a diagonal matrix,  $\bar{\mathbf{Z}}_f$  is a lower triangular matrix, and  $\bar{\mathbf{Z}}_b$  is an upper triangular matrix, all built from  $\bar{\mathbf{Z}}_2$  ( $\bar{\mathbf{Z}}_2 = \bar{\mathbf{Z}}_f + \bar{\mathbf{Z}}_d + \bar{\mathbf{Z}}_b$ ). The subscripts  $d$ ,  $f$ , and  $b$  stand for diagonal, forward, and backward matrices but are referred to, respectively, as diagonal, lower, and upper triangular matrices. Moreover, the unknown vector is decomposed as  $\mathbf{U} = \mathbf{U}_f + \mathbf{U}_b$ , in which  $\mathbf{U}_f$  gives the forward contribution, which starts from the current element  $(x_1, y_1)$  and moves from left to right and then from top to bottom until reaching the receiving element  $(n, m)$ , and  $\mathbf{U}_b$  gives the backward contribution (from right to left and then from bottom to top), as illustrated in Fig. 1.

To compute  $\mathbf{U}$ , an iterative procedure is applied. Assuming first that  $\mathbf{U}_b = 0$ , Eq. (11) is solved for  $\mathbf{U}_f$ . Then substituting  $\mathbf{U}_f$  in Eq. (12),  $\mathbf{U}_b$  is found. The first iteration  $\mathbf{X}_2^{(0)}$  is then equal to  $\mathbf{U}_f + \mathbf{U}_b$ . The scheme is repeated to calculate the next iterations  $\mathbf{U}^{(p)}$  up to the order  $p = P_{\text{FB}}$ .

To validate the E-PILE method, it is tested for the case of a PC plate above a PC Gaussian rough surface. To quantify the convergence of E-PILE versus its

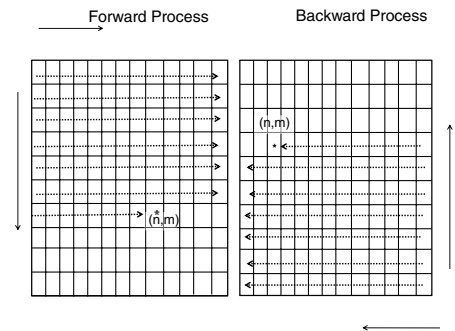


Fig. 1. Forward and backward processes.

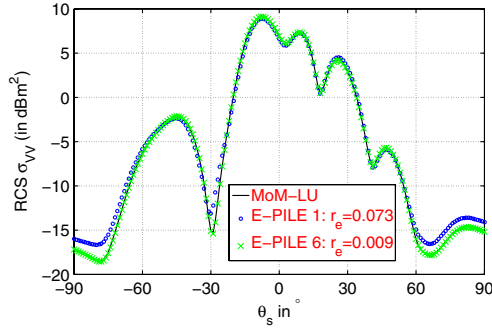


Fig. 2. (Color online) Convergence of E-PILE for the case of plate  $S_1$  above a rough surface  $S_2$ . The parameters are  $S_1 = 2\lambda_0 \times 2\lambda_0$ ,  $S_2 = 6\lambda_0 \times 6\lambda_0$ , the distance between them  $D = 4\lambda_0$ , RMS height  $\sigma_z = 0.2\lambda_0$ , correlation lengths  $L_{cx} = L_{cy} = 1\lambda_0$ , and the incidence angle  $\theta_i = 0^\circ$ .

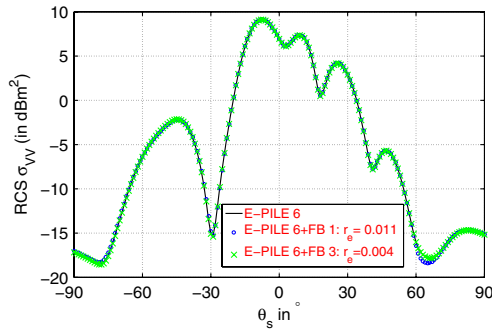


Fig. 3. (Color online) Convergence of E-PILE + FB for the same scenario as in Fig. 2.

order, the relative residual error ( $r_e = \text{norm}(\mathbf{X} - \mathbf{X}_{LU}) / \text{norm}(\mathbf{X}_{LU})$ ) is introduced. In Fig. 2, the scattering coefficient computed from the E-PILE method is compared with that obtained from the MoM-LU versus the scattering angle. From the results, we can notice that as the order of E-PILE increases, it converges to LU (E-PILE 6 is closer to LU than E-PILE 1). Other simulations, not depicted here, showed that E-PILE converges more quickly as the distance between the two plates increases, since the electromagnetic coupling is less important than the previous case. In fact, the order  $P_{E-PILE}$  is directly related to the coupling between the two scatterers. Figure 3 shows the convergence of E-PILE + FB versus its order. To obtain the order  $P_{E-PILE}$ , which permits us to have a good convergence,  $r_e$  must be smaller than a chosen value equal to  $10^{-2}$ , so E-PILE 6 is chosen to be the reference method in Fig. 3. From the results, we can notice that as the order of FB increases, it converges to E-PILE 6. Figure 4 compares the computing time of the two algorithms (MoM-LU) and (E-PILE 6 + FB 3). From the results, we can notice that the E-PILE + FB algorithm allows us to reduce significantly the computing time.

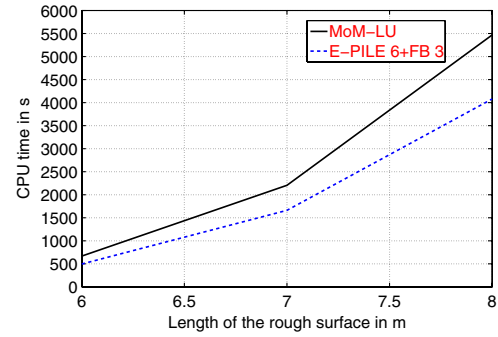


Fig. 4. (Color online) Comparison of the computing time of the two methods versus the length of the rough surface.

In conclusion, a new efficient rigorous method to study the electromagnetic scattering from a 3D problem of two scatterers is presented. The method is based on the rigorous E-PILE method, originally developed to predict the field scattering from an object above a 1D rough surface. In this Letter, the E-PILE method is extended to solve 3D electromagnetic problems. A major advantage of the E-PILE method is that it can be combined with algorithms originally developed to solve problems of scattering from a single scatterer in free space. Consequently, we combined the E-PILE method with the FB method to calculate the local interactions on the rough surface; by this the complexity of the method is reduced significantly. Other advantages of E-PILE + FB are that it is a rigorous fast method dealing with large 3D problems, that it has a large validity domain that makes it able to handle most of the configurations studied in the literature, and that it allows us to study new ones without restrictions to small roughness.

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