J. of Electromagn. Waves and Appl., Vol. 24, 1195–1205, 2010

EXTENSION OF THE ROUGHNESS CRITERION OF A ONE-STEP SURFACE TO A ONE-STEP LAYER

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Abstract—The problem of electromagnetic wave scattering from a one-step surface is a well-known subject. The reflected power can be evaluated using the widely-used Rayleigh roughness parameter. Here, its extension to the case where the one-step surface overlies a perfectly flat surface, called one-step layer, is studied.

1. INTRODUCTION

The Rayleigh roughness criterion, first studied by Lord Rayleigh [1, 2], is a common tool for estimating the degree of roughness of a rough surface. It is a qualitative approach which makes it possible to calculate the phase variations of the reflected wave of a rough surface [3]. For the simple case of a one-step surface, it allows one to calculate the phase difference between two reflected waves of a one-step surface, by neglecting edge diffraction phenomenon [4], and by considering geometric rays [5, 6]. In other words, the illuminated surface area must be much greater than the electromagnetic wavelength.

In what follows, the case of a one-step surface is recalled, and a roughness criterion associated to an attenuation of the reflected power in the specular direction by a factor 4 is defined. This provides one a criterion for estimating the attenuation of the power received by an antenna owing to the presence of a step in a surface. Then, this approach is extended to the case where the one-step surface overlies a perfectly flat surface (called here one-step layer).

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2. CASE OF A ONE-STEP SURFACE

2.1. General Description

Let us consider an incident plane wave inside a medium Ω_1 of wavenumber k_1 and of incidence angle θ_i. For the case of a one-step surface (see Figure 1), the wave reflected by the surface does not have the same path way if the wave is incident on the surface at a point on the left or on the right of the step. For a surface point A_1 (of height $\zeta = \zeta_A$) on the left of the step, the reflected field E_1 has a phase denoted ϕ_1, and for a surface point A'_1 (of height $\zeta = \zeta_{A'}$) on the right of the step, the reflected field E'_1 has a phase denoted ϕ'_1 (see Figure 1).

We consider here an incident wave that impinges an area S of the surface Σ_A in which the step is present, and such that the illuminated areas on the left and on the right of the step, S_A and $S_{A'}$, respectively, are equal: $S_A = S_{A'} = S/2$. S is such that $S = L \times b$, with L and b the x- and y-components of S, respectively. The condition $\{L,\ b\} \gg \lambda_1$ holds (with λ_1 the incident wavelength), so that the surface can be considered of infinite area. These two wave groups of respective phases ϕ_1 and ϕ'_1 interfere to form the total reflected field E_r, the interference being described by the phase difference $\delta\phi = \phi_1 - \phi'_1$. By noting $\delta\zeta_A = \zeta_A - \zeta_{A'}$ the step height, it is easy to show that $\delta\phi$ is given by:

$$\delta\phi = 2k_1 \delta\zeta_A \cos\theta_i. \tag{1}$$

Thus, it is usually said that if the phase difference $|\delta\phi| < \pi/2$, the waves interfere constructively, and the step surface can be considered as slightly rough, or even nearly flat if $|\delta\phi| \ll \pi/2$. In the reverse configuration, if $|\delta\phi| > \pi/2$ (and $|\delta\phi| < 3\pi/2$), the waves interfere destructively, and the surface can be considered as moderately rough or even very rough. In what follows, a slightly different definition, which considers the total reflected power P_r, is proposed.

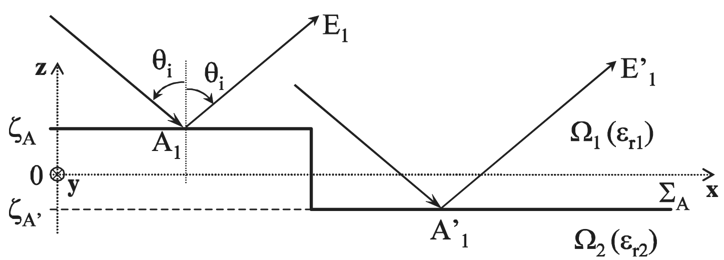

Figure 1. Phase difference between two reflected waves E_1 and E'_1 of a one-step surface: representation in the plane $(\mathbf{x}, \mathbf{z})$.

2.2. Power Roughness Criterion

Let us focus on the determination of the total reflected power P_r from such a surface, comparatively to the case of a perfectly flat surface. The incident power P_i is given by the relation

$$P_i = |E_i|^2 S \cos\theta_i/(2Z_1), \tag{2}$$

with $|E_i|$ the incident field modulus and Z_1 the incident wave impedance.

For the case of a perfectly flat interface, the total reflected power is maximum, $P_r \equiv P_r^{\max}$, and is easily obtained, as all the reflected fields E_r are in phase (interfere constructively), $\delta\phi$=0. Thus, the total reflected power P_r, given with respect to the incident power P_i by the general relation

$$P_r/P_i = |E_r|^2/|E_i|^2, \tag{3}$$

can be simplified in this simple case of a flat surface, $|E_r|$ being given by $|E_r| = |r_{12}(\theta_i)E_i|$, with r_{12} the Fresnel reflection coefficient inside Ω_1 and onto Ω_2.

For the case of a one-step surface, the total reflected field E_r results from the coherent summation of the reflected fields E_1 at the surface of height ζ_A on the left of the step and of the reflected fields E_1' at the surface of height $\zeta_{A'}$ on the right of the step. Under the condition $S_A = S_{A\prime} = S/2$, E_r is given by the relation

$$E_r = E_1 + E_1' = |E_1| \left(e^{j\phi_1} + e^{j\phi_1'}\right) = |E_1| \left(1 + e^{j\delta\phi}\right). \tag{4}$$

As a consequence, the total reflected power P_r, given by Equation (3), is maximum if E_1 and E_1' are in phase, i.e., $\delta\phi = 0$ in Equation (1) (or more generally, $\delta\phi = 2k\pi$, with k integer), corresponding to $P_r \equiv P_r^{\max}$. Quite the reverse, $E_r = 0 \Rightarrow P_r = 0$ if E_1 and E_1' are in phase opposition, i.e., $\delta\phi = \pm\pi$ in Equation (1) (or more generally, $\delta\phi = \pi + 2k\pi$, with k integer). This is illustrated in Figure 2 for various values of the phase difference: $\delta\phi = \{+30; +90; +120; +150\}$ degrees, in the case $S_A = S_{A'} = S/2$. As can be seen, when the phase difference increases from $\delta\phi = 30$ degrees to 150 degrees, the total reflected field modulus $|E_r|$ decreases. Note that for $\delta\phi = 120$ degrees, $|E_r| = (|E_1| + |E_{1'}|)/2 = |E_1|$, which is equal to half its maximum value that occurs for a flat surface: $|E_r| = |E_r^{\max}|/2$.

A roughness criterion on the total reflected power P_r, for defining the limit over which the surface is considered as rough, can be expressed as follows: It is defined as the limit roughness for which the total reflected power P_r remains superior to the fourth of its maximum value, equal to the one for the flat case $P_r^{\max}$,

$$P_r \geq P_r^{\max}/4. \tag{5}$$

This corresponds to the criterion on the field modulus $|E_r| \geq |E_r^{\max}|/2$, which is valid for $|\delta\phi| \leq 2\pi/3$. Thus, for the case where the illuminated areas are equal, $S_A = S_{A'} = S/2$ (case illustrated in Figure 2), this leads in Equation (1) to the relation $|\delta\phi| \leq 2\pi/3$ (or more generally, $2k\pi - 2\pi/3 \leq \delta\phi \leq 2k\pi + 2\pi/3$, with k integer). Then, the following condition on the step height $\delta\zeta_A$ holds:

$$|\delta\zeta_A|/\lambda_1 \times \cos\theta_i \leq 1/6. \quad (6)$$

As a consequence, the power roughness criterion, given by the ratio $|\delta\zeta_A|/\lambda_1$, increases with increasing incidence angle θ_i.

3. EXTENSION TO A ONE-STEP LAYER

In this section, the power roughness criterion is extended to the case of a one-step surface over a perfectly flat surface, called one-step layer. When a perfectly flat lower interface is present, multiple reflections inside the inner medium Ω_2 occur, and consequently multiple reflections back into Ω_1 (see Figure 3). Then, the incident wave, which impinges the upper one-step surface, undergoes multiple reflections inside Ω_2. On the left of the step, the thickness of Ω_2 is equal to $\zeta_A - \zeta_B$ and is denoted h, and on the right of the step, the thickness of Ω_2 is equal to $\zeta_{A'} - \zeta_B$ and is denoted h' (see Figure 3).

Here, it is assumed that the inner medium thicknesses, $h = \zeta_A - \zeta_B$ and $h' = \zeta_{A'} - \zeta_B$, are much lower than the length of the illuminated surfaces $L_A = L_{A'}$, so that the intermediate configuration for which the incident wave hits the surface on the left of the step, and some of the reflections back into Ω_1 occur at a point of Σ_A on the right of the step (which occurs for non-normal incidence, $\theta_i \neq 0$), can be neglected (for instance, see Figure 4 for the case where the second reflection occurs on the right of the step). Thus, only the two extreme configurations represented in Figure 3 contribute to the total reflected field E_r.

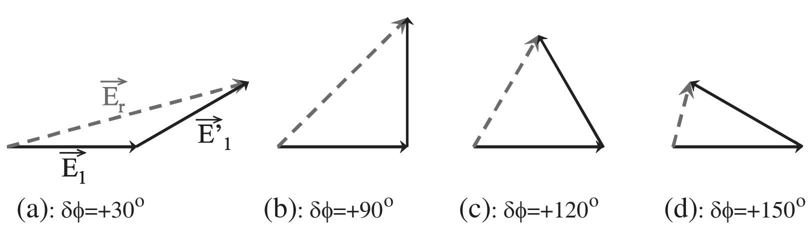

Figure 2. Representation of the reflected field vectors $\vec{E}_1$ and $\vec{E}'_1$, together with the total reflected field vector $\vec{E}_r$ in the case $S_A = S_{A'} = S/2$, for different values of the phase difference $\delta\phi$.

Then, it can be seen that these two extreme configurations correspond to two classical Fabry-Pérot interferometers of thicknesses h and h'. The field $E^{FP}(H)$ reflected by a Fabry-Pérot interferometer of general thickness H is expressed with respect to the incident field E_i by $|E^{FP}(H)| = |r^{eq}(\theta_i; H)E_i|$, with r^{eq} the equivalent Fresnel reflection coefficient given by the relation [7–9]

$$r^{eq}(\theta_i; H) = \left[r_{12}(\theta_i) + r_{23}(\theta_m)\exp\left(-j\phi^{FP}\right)\right] / \left[1 + r_{12}(\theta_i)\, r_{23}(\theta_m)\exp\left(-j\phi^{FP}\right)\right], \tag{7}$$

with ϕ^{FP} the phase difference between two successive reflected fields (for instance between E_1 and E_2 in Figure 3), given by $\phi^{FP} = 2k_2 H\cos\theta_m$. k_2 is the wavenumber inside Ω_2, and θ_m the propagation angle inside Ω_2, given by the Snell-Descartes refraction law $k_1\sin\theta_i = k_2\sin\theta_m$.

As a consequence, for the case on the left of the step in Figure 3, the total reflected field $E^{FP}(h)$ is given by $|E^{FP}(h)| = |r^{eq}(\theta_i; h)E_i|$, with $h = \zeta_A - \zeta_B$, and for the case on the right of the step in Figure 3, the total reflected field $E^{FP}(h')$ is given by $|E^{FP}(h')| = |r^{eq}(\theta_i; h')E_i|$, with $h' = \zeta_{A'} - \zeta_B$. Finally, the total reflected field E_r, given by Equation (4), is expressed for $L_A = L_{A'} = L/2 \gg h'$ by

$$|E_r| = \left|\left[r^{eq}(\theta_i; h) + r^{eq}\left(\theta_i; h'\right)\right] E_i\right|/2. \tag{8}$$

Following the same way as for the single interface case, the phase difference between the two extreme configurations is given by Equation (1), and the criterion for the total reflected power attenuation by a factor 4, comparatively to the case of two flat interfaces remains the same and is given by Equation (6).

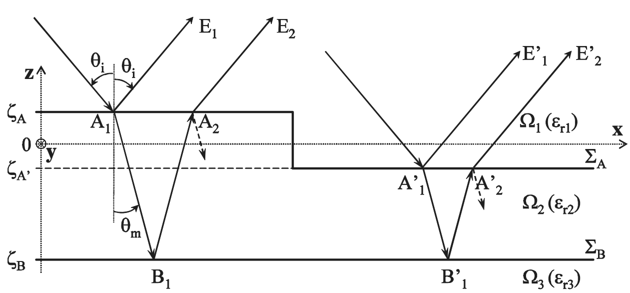

Figure 3. Phase difference between two reflected waves of a one-step surface over a perfectly flat lower interface (one-step layer): representation in the plane $(\mathbf{x}, \mathbf{z})$.

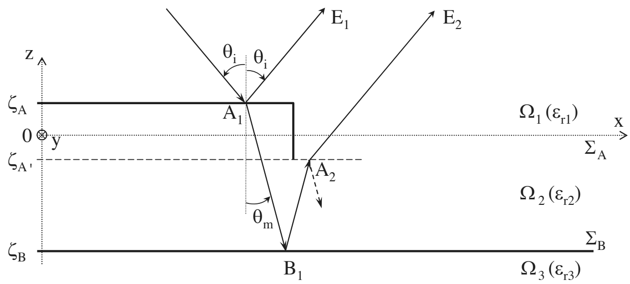

Figure 4. Limit case of the one-step layer, which occurs for non-normal incidence, $\theta_i \neq 0$.

3.1. Case of Two Flat Interfaces

Comparatively to the single flat interface case, the presence of the lower flat interface induces interferences between the multiple reflected fields, so that the total reflected field can significantly vary in modulus between a maximum value and a minimum value which is zero. As a result, comparatively to the single flat interface case, for the simple

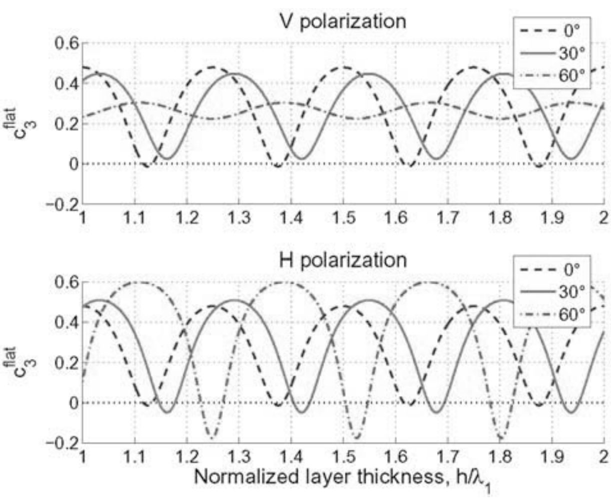

Figure 5. Illustration of the power roughness criterion of two flat interfaces for both V and H polarizations: $c_3^{flat} = |r^{eq}|^2 - |r_{13}|^2/4$ with respect to the normalized layer thickness h/λ_1. The relative permittivities are $\varepsilon_{r2} = 2.2$ and $\varepsilon_{r3} = 80$. The legend represents the values of the incidence angle θ_i in degrees.

case of two flat interfaces, the criterion in Equation (5) simplifies as

$$|r^{eq}(\theta_i; h)|^2 \geq |r_{1x}(\theta_i)|^2/4, \tag{9}$$

r_{1x} being the Fresnel reflection coefficient of the single flat interface case considered depending on the application studied, which can be the interface of the inner medium, $\Omega_x = \Omega_2$, or rather the interface of the lower medium, $\Omega_x = \Omega_3$.

This is illustrated in Figure 5, where the term $c_3^{flat} = |r^{eq}|^2 - 1/4|r_{13}|^2$ is compared to 0 for both V and H polarizations. This term must be ≥ 0 in order to check the power roughness criterion (5). The relative permittivities are $\varepsilon_{r2} = 2.2$ and $\varepsilon_{r3} = 80$. It can be seen that the term c_3^{flat} is always positive for both polarizations and for all plotted incidence angles θ_i. This is due to the fact that $\varepsilon_{r2} = 2.2$ is close to $\varepsilon_{r1} = 1$ and much inferior to $\varepsilon_{r3} = 80$, so that the main contribution to the total reflected field E_r comes from the second-order reflected field E_2 (see Figure 3). The other contributions (E_1, E_3, E_{r4}, etc.) are low comparatively to E_2, so that the coherent summation of all contributions E_n (with $n = \{1, 2, 3, 4, \dots\}$) remains always positive. Thus, for this typical configuration, the Rayleigh power roughness criterion (5) is validated for moderate incidence angles.

The influence of the relative permittivities ε_{r2} and ε_{r3} can then be studied. First, by changing only the inner relative permittivity ε_{r2}, only the term $|r^{eq}|^2$ inside $c_3^{flat} = |r^{eq}|^2 - 1/4|r_{13}|^2$ is modified. As illustrated in Figure 6 for $\varepsilon_{r2} = 4$, increasing ε_{r2} implies an increase of

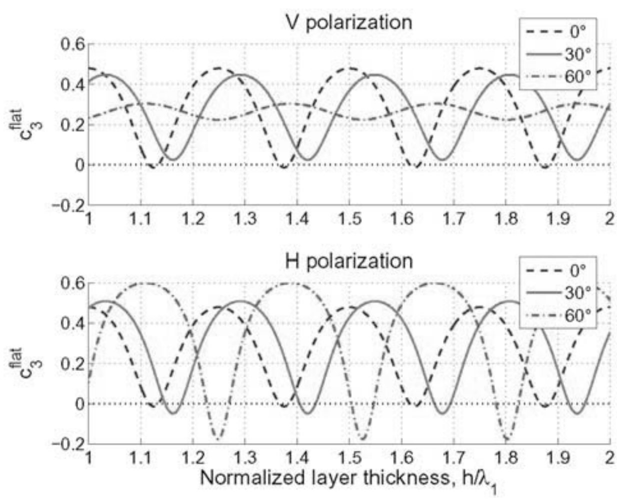

Figure 6. Same configuration as in Figure 4, except for the inner medium relative permittivity $\varepsilon_{r2} = 4$.

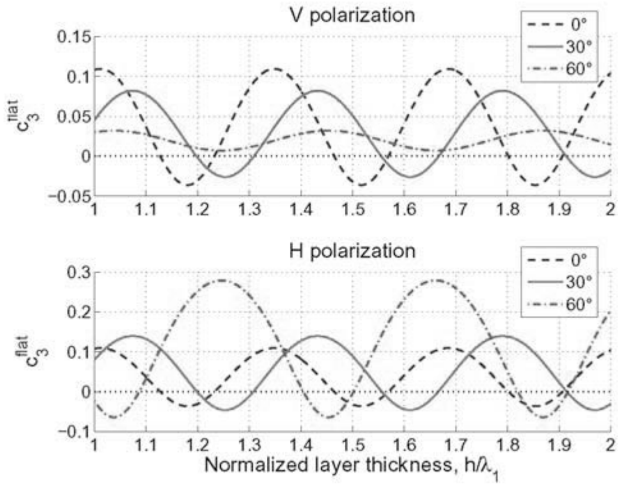

Figure 7. Same configuration as in Figure 4, except for the lower medium relative permittivity $\varepsilon_{r3} = 5$.

the phase term ϕ^{FP} because the latter is proportional to $(\varepsilon_{r2})^{1/2}$, so that the oscillation frequency of c_3^{flat} with respect to the normalized layer thickness h/λ_1 is increased. Moreover, by increasing ε_{r2} from 2.2 to 4, the second-order reflected field E_2 is decreased and the first-order one E_1 is increased, so that they tend to have contributions of the same order. Thus, the oscillations amplitude is increased, and the mean level of the oscillations is decreased and gets closer to 0. As a consequence, for a few configurations the power roughness criterion (5) is not checked any more.

Second, by changing only the lower relative permittivity ε_{r3}, both terms $|r^{eq}|^2$ and $|r_{13}|^2$ inside c_3^{flat} are modified. This time, the oscillation frequency with respect to the normalized layer thickness h/λ_1 remains constant, as ϕ^{FP} does not depend on ε_{r3}. This is illustrated in Figure 7 for $\varepsilon_{r3} = 5$ (by keeping $\varepsilon_{r2} = 2.2$). Nevertheless, a significant decrease of ε_{r3} implies a significant decrease of the term $|r^{eq}|^2$. Indeed, this does not change the first-order reflected field E_1 but significantly decreases the second-order reflected field E_2, so that E_2 has this time a lower contribution than E_1. As a result, comparatively to Figure 5, the oscillations amplitude is decreased, and the mean level of the oscillations is significantly decreased and gets much closer to 0. It can be deduced that the term $|r_{13}|^2$ also significantly decreases at the same time, which contributes to remain the mean level of c_3^{flat} close to zero but still positive. As a consequence, similarly as in Figure 6, for some configurations the power roughness criterion (5) is not checked any more.

In next subsection, the influence of the one-step layer is studied, and in particular the influence of the step height $\delta\zeta_A$.

3.2. Case of a One-step Layer

Comparatively to the single flat interface case, the case under study has two constraints: One on the step height $\delta\zeta_A$ given by Equation (6), and one on the Fabry-Pérot interferometer configuration given by Equation (9). As a consequence, comparatively to the single flat interface case, for the case under study, the criterion in Equation (5) simplifies for $L/2 \gg h'$ as

$$|r^{eq}(\theta_i; h) + r^{eq}(\theta_i; h')|^2/4 \geq |r_{1x}(\theta_i)|^2/4, \tag{10}$$

with $h' = h - \delta\zeta_A$. This is illustrated in Figure 8 with $\Omega_x = \Omega_3$, where the term $c_3^{step} = |r^{eq}(\theta_i; h) + r^{eq}(\theta_i; h')|^2/4 - |r_{13}(\theta_i)|^2/4$ is compared to 0 for both V and H polarizations. Once again, this term must be ≥ 0 in order to check the power roughness criterion (5). The normalized step height is $\delta\zeta_A/\lambda_1 = 0.1$, and the other parameters are the same as in Figure 5 for the case of two flat interfaces.

Contrary to the case of two flat interfaces in Figure 5, for all plotted cases the term c_3^{step} takes values inferior to 0, except in V

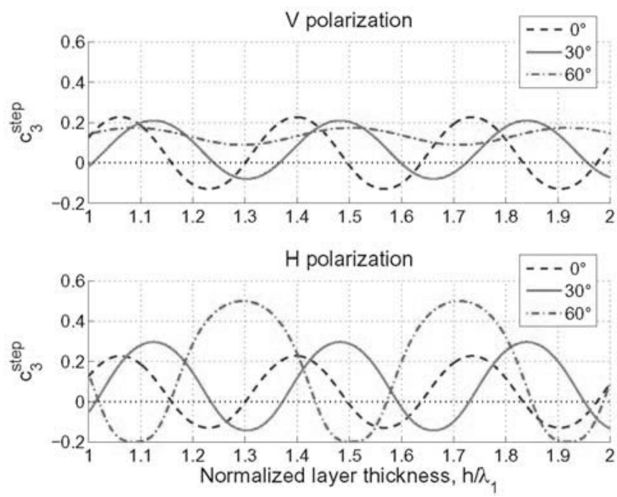

Figure 8. Same configuration as in Figure 4, but for a one-step layer with a normalized step height $\delta\zeta_A/\lambda_1 = 0.1$.

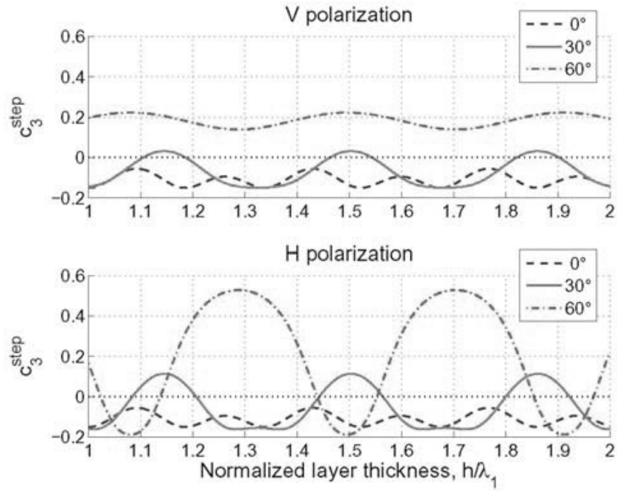

Figure 9. Same configuration as in Figure 7, except for the normalized step height $\delta\zeta_A/\lambda_1 = 0.5$.

polarization for $\theta_i = 60°$. This illustrates that the presence of a one-step in a surface plays a significant role in the power roughness criterion, even for relatively low normalized step heights $\delta\zeta_A/\lambda_1$. Moreover, as illustrated above for the case of two flat interfaces, the influence of the relative permittivities ε_{r2} and ε_{r3} is also significant.

The influence of changing the step height $\delta\zeta_A$ is illustrated in Figure 9 where $\delta\zeta_A/\lambda_1 = 0.5$ (comparatively to Figure 8 where $\delta\zeta_A/\lambda_1 = 0.1$). This simulation confirms that the step height $\delta\zeta_A$ has a significant influence on the total reflected power, because significant differences appear with Figure 8. In particular, for low to moderate incidence angles θ_i, secondary interferences can be observed (here mainly for $\theta_i = 0$), which can be attributed to the interference between the fields reflected from two flat interfaces with thicknesses h and h'. Moreover, it can be seen that contrary to Figure 8, for $\theta_i = 0°$ and in both polarizations, the power roughness criterion is never checked which means that in these two cases the one-step layer is always very rough. This must be compared with $\theta_i = 60°$ in V polarization, where the power roughness criterion is always checked which means that in this case the one-step layer is always slightly rough. This illustrates the great variability of the electromagnetic roughness of the one-step layer for this case.

Thus, the presence of a flat interface under a one-step surface has a significant influence on its electromagnetic roughness, leading to specific physical behaviours.

4. CONCLUSION

In conclusion, from the case of a one-step surface, the electromagnetic roughness of a one-step layer was derived. It was then established that for a one-step layer, the electromagnetic roughness significantly varies with several parameters. Thus, depending on the media relative permittivities, the layer thickness, the step height, the incidence angle, and the polarization, the layer can be considered as slightly rough, or on the contrary very rough.

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