Effect of Correlation Between Shadowing and Shadowed Points on the Wagner and Smith Monostatic One-Dimensional Shadowing Functions

Christophe Bourlier, Joseph Saillard, and Gérard Berginc

Abstract—**The Wagner [1] and Smith [2], [3] classical monostatic one-dimensional (1-D) shadowing functions assume that the joint probability density of heights and slopes is uncorrelated, thus inducing an overestimation of the shadowing function. The goal of this article is to quantify this assumption. More recently, Ricciardi and Sato [4], [5] proved that the shadowing function is given rigorously by Rice's infinite series of integrals. We observe that the approach proposed by Wagner retains only the first term of this series, whereas the Smith formulation uses the Wagner model by introducing a normalization function. In this article, we first calculate the shadowing function based on the Ricciardi and Sato work for an uncorrelated process. We will see that the uncorrelated results do not have any physical sense. Next, the Wagner and Smith formulations will be modified in order to introduce the correlation. Correlated and uncorrelated results are compared with the reference solution, which is determined by generating a surface [8] for a Gaussian autocorrelation function. So, we will show that the correlation improves the results for values $\mu \leq 2\sigma$, where μ represents the slope of incident ray and σ the slopes variance of the surface. Finally, our results will be compared to those given in [9], determined from the first three terms of Rice's series, but the shadowing function used is not averaged over the slopes.**

Index Terms—**Electromagnetic scattering by rough surfaces.**

I. Introduction

THE monostatic shadowing function characterizes the surface fraction which is visible by an observant. Work on the shadowing function has been going on since the 1960's in order to determine the electromagnetic scattering from a randomly rough surface. The energy scattered from the total surface is multiplied by the shadowing function [6]. The analytical shadowing function proposed by Beckmann [7] is equal to the illuminated portion of the surface and it varies from one at normal incidence to zero at grazing angle. Brokelman and Hagfors [8] suggested that a shadowing function is equal to the fraction specular points that are illuminated rather than the fraction of all surface points. They showed that the analytical function proposed by Beckman is accurate for grazing and quasi-normal angles of the surface, whereas there is a difference in their results between those two boundaries.

For an observation length L, the shadowing function $S(\theta, F, L)$ is equal to the probability that the point $F(\xi_0, \gamma_0)$ on a random rough surface of given ξ_0 height above the mean plane and with local slope $\gamma_0 = \partial z/\partial y$ is illuminated, when the surface is crossed by a beam incident from direction θ Fig. 1. It is equal [1]–[3] to

$$S(\theta, F, L) = \Upsilon(\mu - \gamma_0) \cdot \exp\left[-\int_0^L g(\theta|F; l)\, dl\right] \quad \text{with}$$
$$\Upsilon(\mu - \gamma_0) = \begin{cases} 0 & si \quad \gamma_0 \geq \mu \\ 1 & si \quad \gamma_0 < \mu \end{cases} \tag{1}$$

where $g(\theta|F; l)\, dl$ is the conditional probability that the beam intersects the surface in the interval $[l; l + dl]$, given the ray does not cross the surface in the interval $[0; l]$, Υ is the Heaviside function and $\mu = \cot\theta$ denotes the slope of incident beam. Therefore, the averaged shadowing function over the slopes γ_0 and over the heights ξ_0 is given by

$$S(\theta, L) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\theta, \{\xi_0, \gamma_0\}, L) \cdot p(\xi_0, \gamma_0)\, d\xi_0\, d\gamma_0 \tag{2}$$

where $p(\xi_0, \gamma_0)$ is the joint probability density of heights and slopes. In [9], Kapp and Brown only determine the integration over ξ_0. Substituting (1) into (2), we obtain

$$S(\theta, L) = \int_{-\infty}^{\infty} \int_{-\infty}^{\mu} p(\xi_0, \gamma_0) \cdot \exp\left[-\int_0^L g(\theta|F; l)\, dl\right] d\xi_0\, d\gamma_0. \tag{3}$$

Ricciardi and Sato [4], [5] showed that the function is given by Rice's infinite series of integrals (Section I). Wagner retains only the first term of the series, whereas Smith uses the Wagner formulation combined with a normalization function (Section III). Moreover, they assume that the process is Gaussian and uncorrelated. The Ricciardi and Sato expression is calculated assuming an uncorrelated Gaussian process and we show that the analytical solution obtained has no physical meaning. Next, the Wagner and Smith formulations are used in order to introduce the correlation. We show that the shadowing functions are always given in function of only one parameter $v = \mu/(\sigma\sqrt{2})$ where μ represents the slope of incident beam, and σ^2 the slopes variance of the surface. But they also depend on the autocorrelation function and its first and second derivatives. So we can say that the autocorrelation function is a constant if we neglect the correlation.

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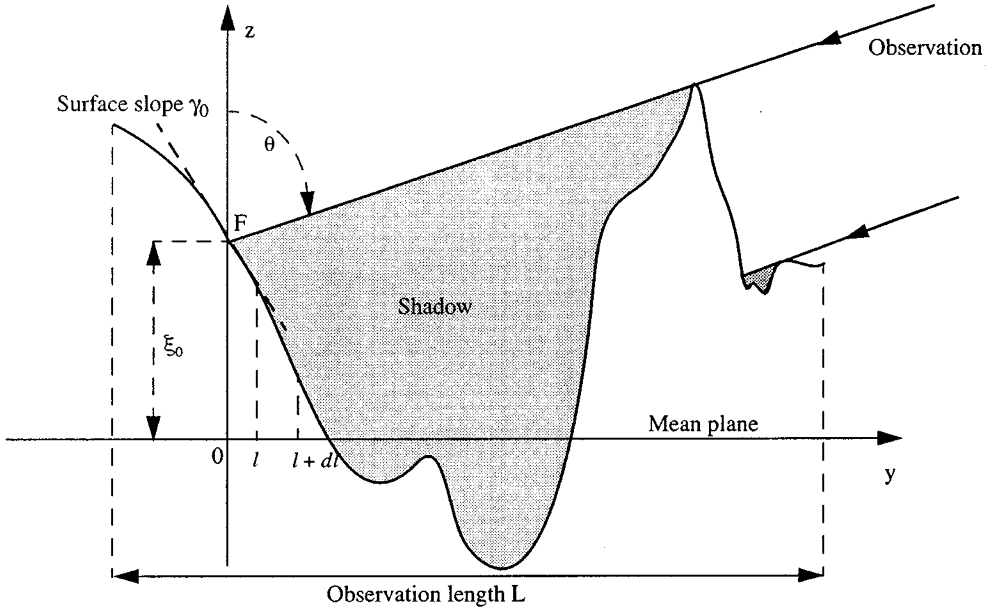

Fig. 1. The monostatic configuration of the shadowing function.

In order to estimate this hypothesis, the Wagner and Smith classical shadowing functions and those calculated with the correlation, are compared with the reference solution obtained by generating a Gaussian surface (Section IV). We show that the effect of the correlation improves the results and it is negligible when the parameter v is greater than 1.4. Finally, our results will be compared to those given in [9], found from the first three terms of Rice's series, but their shadowing function is not averaged over the slopes.

II. Sato and Ricciardi Approaches

Ricciardi and Sato [4], [5] show that the function g is given by an infinite series

$$\begin{aligned} g(\theta|F;l) =& W_1(l|F) - \int_0^t W_2(l_1,l|F)\,dl_1 \\ &+ \int_0^l dl_1 \int_{l_1}^l W_3(l_2,l_1,l|F)\,dl_2 \\ &- \cdots + (-1)^{n-1} \int_0^l dl_1 \int_{l_1}^l dl_2 \\ &\cdots \int_{l_{n-1}}^l W_n(l,l_1,\cdots,l_{n-1}|F)\,dl_{n-1} \end{aligned} \tag{4a}$$

with

$$\begin{aligned} &W_n(l,l_1,\cdots,l_{n-1}|F) \\ &= \int_\mu^\infty d\gamma_1 \int_\mu^\infty d\gamma_2 \cdots \int_\mu^\infty d\gamma_n \\ &\quad \cdot \prod_{i=1}^n (\gamma_i - \mu) p_{2n+2}(\vec{S};\vec{G}|\xi_0,\gamma_0) \end{aligned} \tag{4b}$$

where $W_n(l,l_1,\cdots,l_{n-1}|F)\ dl_1\ \ dl_2\cdots dl_{n-1}$ is the joint probability that the incident ray of equation $S_n = \xi_0 + \mu l_n$ crosses the surface $\xi(l_n)$, with a slope μ inferior to the slope γ_n surface's abscissa l_n in the intervals $\{[l_1;l_1+dl_1], [l_2;l_2+dl_2],\cdots,[l_{n-1};l_{n-1}+dl_{n-1}]\}$, conditionally to the knowledge of $F(\xi_0,\gamma_0)$. $p_{2n+2}(\vec{S};\vec{G}|\xi_0,\gamma_0)$ is the joint probability density of vectors $\vec{S}^T = [S_1,S_2,\cdots,S_n]$ and $\vec{G}^T = [\gamma_1,\gamma_2,\cdots,\gamma_n]$ at abscissa points $\{l_1,l_2,\cdots,l_n\}$, knowing $\{\xi_0,\gamma_0\}$. The problem is slightly different from these presented by [4] and [5] because the probability density p_{2n+2} is conditioned in our case by the variables $\{\xi_0,\gamma_0\}$, whereas [4] and [5] only consider the term ξ_0.

For an uncorrelated Gaussian stationary process and with zero mean $p_{2n+2}(\vec{S};\vec{G}|\xi_0,\gamma_0)$ is given by

$$\begin{aligned} &p_{2n+2}(\vec{S};\vec{G}|\xi_0,\gamma_0) \\ &\quad = \frac{1}{(2\pi\sigma\omega)^n} \exp\left[-\frac{1}{2}\sum_{i=1}^n \left(\frac{\gamma_i^2}{\sigma^2} + \frac{S_i^2}{\omega^2}\right)\right] \end{aligned} \tag{5}$$

where $\{\sigma^2,\omega^2\}$ are the variance of slopes and heights respectively. Substituting (5) into (4b), W_n becomes

$$\begin{aligned} W_n =& \exp\left(-\sum_{i=1}^n \frac{S_i^2}{2\omega^2}\right) \\ &\cdot \left[\frac{1}{2\pi\sigma\omega}\int_\mu^\infty (\gamma-\mu)\exp\left(-\frac{\gamma^2}{2\sigma^2}\right)\,d\gamma\right]^n. \end{aligned} \tag{6}$$

We obtain after integration over γ

$$W_n = \exp\left(-\sum_{i=1}^n \frac{S_i^2}{2\omega^2}\right)\left(\frac{\mu\Lambda}{\omega\sqrt{2\pi}}\right)^n \quad \text{with}$$
$$v = \frac{\mu}{\sigma\sqrt{2}} \quad \Lambda = \frac{e^{-v^2} - v\sqrt{\pi}\,\mathrm{erfc}(v)}{2v\sqrt{\pi}} \tag{7}$$

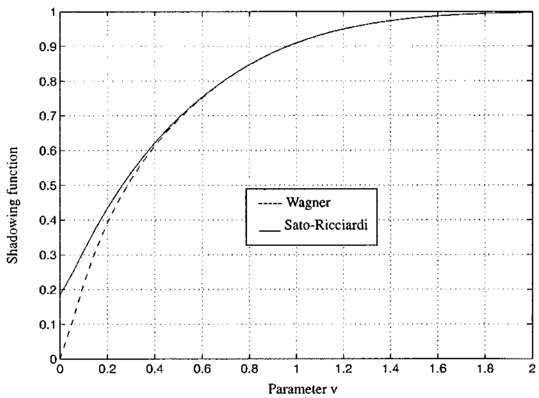

Fig. 2. Wagner and Sato–Ricciardi uncorrelated shadowing functions.

with erfc the complementary error function. Substituting (7) into (4) and doing the different integrations we can write

$$g = g_0 \cdot \exp(-X) \quad \text{with} \quad \cdot \begin{cases} g_0 = \left(\dfrac{\mu\Lambda}{\omega\sqrt{2\pi}}\right) \exp\left[-\dfrac{\xi_0+\mu l)^2}{2\omega^2}\right] \\ X = \dfrac{\Lambda}{2}\left[\operatorname{erf}\left(\dfrac{\xi_0+\mu l}{\omega\sqrt{2}}\right) - \operatorname{erf}\left(\dfrac{\xi_0}{\omega\sqrt{2}}\right)\right]. \end{cases} \tag{8}$$

Using relations (8) and (1), the shadowing function integrated for an infinite length is given by

$$\begin{aligned} S_{SR}(\theta, F) &= S(\theta, F, L \to \infty) \\ &= \Upsilon(\mu-\gamma_0)\exp\left\{\exp\left[-\frac{\Lambda}{2}\operatorname{erfc}\left(\frac{\xi_0}{\omega\sqrt{2}}\right)\right] - 1\right\}. \end{aligned} \tag{9}$$

Substituting (9) into (2), the averaged shadowing function is

$$S_{SR}(v) = \left[1 - \frac{\operatorname{erfc}(v)}{2}\right]\left[\frac{E_1(-e^{-\Lambda}) - E_1(-1)}{\Lambda e^1}\right] \quad \text{with}$$
$$E_1(x) = \int_1^\infty \frac{e^{-st}}{t}\, dt. \tag{10}$$

The Wagner shadowing function [1] obtained from [4] and [5] first term is given by

$$S_W(v) = \left[1 - \frac{\operatorname{erfc}(v)}{2}\right]\left(\frac{1-e^{-\Lambda}}{\Lambda}\right). \tag{11}$$

Fig. 2 represents the Wagner ((10)) and Sato–Ricciardi ((11)) functions. We observe an identical behavior of these two curves for $v \geq 0.6$; on the other hand, it differs for lower values corresponding to grazing incidence angles and we can write

$$S_{SR}(0) = e^{-1}/2 = 0{,}184 \quad \text{and} \quad S_W(0) = 0. \tag{12}$$

Physically, the shadowing function is equal to zero at the grazing angle $\theta = 90°$, which involves that when the correlation is not introduced, the Sato and Ricciardi uncorrelated results are not correct at grazing angles. On the other hand, the Wagner result is accurate but overestimates the shadowing function. The phenomenon modeling is not being accurate enough,

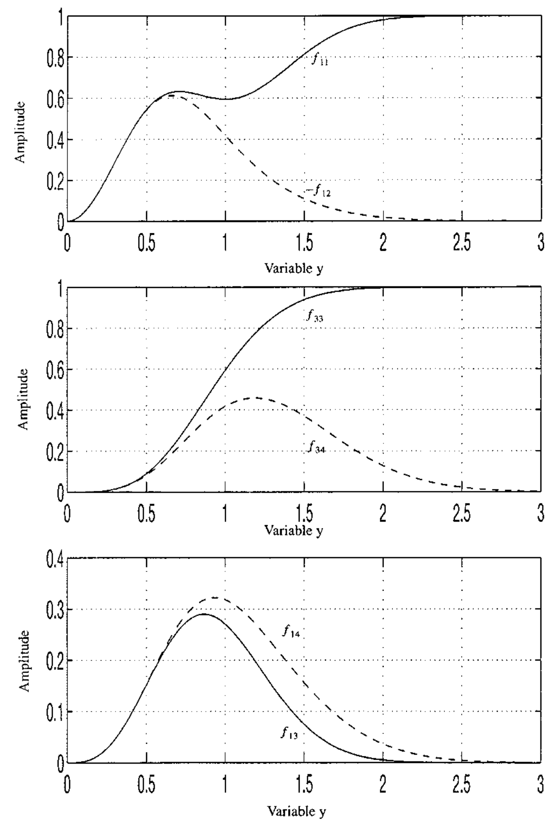

Fig. 3. Functions f_{ij} for a Gaussian autocorrelation function.

it is essential to include the correlation. Unfortunately, the complexity of (4) makes the analytical determination of function g very difficult, indeed impossible. Nevertheless, the analytical calculations are possible for Wagner's and Smith's correlated functions.

III. Wagner and Smith Approaches with Correlation

The Wagner function [1] is given by the first term $W_1(l|F)$ of the series (4)

$$g_W(\theta|F;l) = W_1(l|F) = \int_\mu^\infty (\gamma_1 - \mu)p_4(\xi_0+\mu l; \gamma_1)|\xi_0)\, d\gamma_1. \tag{13a}$$

Smith [2], [3] uses the Wagner formulation by introducing a normalization function

$$g_S(\theta|F;l) = \frac{\displaystyle\sum_\mu^\infty (\gamma_1-\mu)p_4(\xi_0+\mu l; \gamma_1|\xi_0,\gamma_0)\, d\gamma_1}{\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\xi_0+\mu l} p_4(\xi_1,\gamma_1|\xi_0,\gamma_0,\gamma_0)\, d\xi_1\, d\gamma_1}. \tag{13b}$$

A correlated Gaussian stationary process with zero mean is defined as follows:

$$\begin{aligned} &p_4(\xi_1, \gamma_1|\xi_0, \gamma_0) \\ &\quad = \frac{\sigma\omega}{2\pi\sqrt{M}}\exp\left(-\frac{1}{2}\,\vec{V}^T[C]^{-1}\vec{V} + \frac{\xi_0^2}{2\omega^2} + \frac{\gamma_0^2}{2\sigma^2}\right) \end{aligned} \tag{14a}$$

with

$$\vec{V} = \begin{bmatrix} \xi_0 \\ \xi_1 \\ \gamma_0 \\ \gamma_1 \end{bmatrix} \quad \text{and}$$

$$[C] = \begin{bmatrix} \omega^2 & R_0 & 0 & R_1 \\ R_0 & \omega^2 & -R_1 & 0 \\ 0 & -R_1 & \sigma^2 & -R_2 \\ R_1 & 0 & -R_2 & \sigma^2 \end{bmatrix} \tag{14b}$$

where R_0 is the spatial autocorrelation function assumed even and derivate at zero, $\{R_1, R_2\}$ its first and second derivatives. The heights variance ω^2 is equal to $R_0(0)$ and the slopes variance σ^2 is $-R_2(0)$. M is the determinant of the covariance matrix $[C]$, and $R_1(0) = 0$. Substituting (14b), (14a) into (13a), and perfoming the integration over γ_1, we obtain

$$\begin{aligned} g_W(\theta|F;l) &= \frac{\sigma\omega\ \exp[-D-\mu(\mu A+2B)]}{4\pi A\sqrt{M}} \\ &\cdot \left[1-\sqrt{\pi}\exp\left[\frac{(B+\mu A)^2}{A}\right]\frac{B+\mu A}{\sqrt{A}}\ \text{erfc}\left(\frac{B+\mu A}{\sqrt{A}}\right)\right] \end{aligned} \tag{15a}$$

with (15b), shown at the bottom of the page, and

$$\begin{cases} C_{11} = \omega^2(\sigma^4 - R_2^2) + R_1^2\sigma^2 \\ C_{12} = R_0(R_2^2 - \sigma^4) - R_1^2 R_2 \\ C_{13} = R_1(R_0\sigma^2 - \omega^2 R_2) \end{cases} \cdot \begin{cases} C_{14} = R_1(\omega^2\sigma^2 + R_1^2 - R_0R_2) \\ C_{33} = \sigma^2(R_0^2 - \omega^4) - R_1^2\omega^2 \\ C_{34} = R_2(\omega^4 - R_0^2) + R_1^2 R_0. \end{cases} \tag{15c}$$

Using the same method, the determination of the Smith conditional probability leads to (16a), shown at the bottom of the page, with

$$\begin{cases} A_1 = (C_{11}C_{33} - C_{13}^2)D_1 \quad \dfrac{1}{2C_{33}M_1} = D_1 \\ B_1 = [\xi_0(C_{12}C_{33} + C_{14}C_{13}) \\ \qquad + \gamma_0(C_{13}C_{34} - C_{14}D_{33})]D_1 \\ C_1 = [\xi_0^2(C_{11}C_{33} - C_{14}^2) + \gamma_0^2(C_{33}^2 - C_{34}^2) \\ \qquad + 2\xi_0\gamma_0(C_{13}C_{33} - C_{14}C_{34})]D_1. \end{cases} \tag{16b}$$

The function introduced at the denominator of (16a) represents the Smith normalization. In the Gaussian case, the autocorrelation function is

$$R_o(y) = \omega^2\ \exp(-y^2) \quad \text{with} \quad y = \frac{l}{L_c} \tag{17}$$

where L_c is the correlation length. The first R_1 and second R_2 derivatives are defined as follows:

$$R_1(y) = -\frac{2\omega^2}{L_c}\ ye^{-y^2} \quad \text{and} \quad R_2(y) = -\frac{2\omega^2}{L_c^2}\ (1-2y^2)e^{-y^2}. \tag{18}$$

Substituting (17), (18) into (15c), we obtain

$$\begin{cases} \dfrac{C_{11}}{2M_1} = \dfrac{1}{2\omega^2}\ \dfrac{f_{11}}{f_M} \\ \dfrac{C_{12}}{2M_1} = \dfrac{1}{2\omega^2}\ \dfrac{f_{12}}{f_M} \\ \dfrac{C_{33}}{2M_1} = \dfrac{1}{2\sigma^2}\ \dfrac{f_{33}}{f_M} \\ \dfrac{C_{34}}{2M_1} = \dfrac{1}{2\sigma^2}\ \dfrac{f_{34}}{f_M} \\ \dfrac{C_{13}}{2M_1} = \dfrac{1}{\sqrt{2}\sigma\omega}\ \dfrac{f_{13}}{f_M} \\ \dfrac{C_{14}}{2M_1} = \dfrac{1}{\sqrt{2}\sigma\omega}\ \dfrac{f_{14}}{f_M} \end{cases} \quad \text{with}$$

$$\begin{cases} A = \dfrac{C_{33}}{2M} \quad M = \dfrac{C_{33}^2 - C_{34}^2}{\omega^4 - R_0^2} \\ B = \dfrac{\xi_0 C_{14} - \xi_1 C_{13} + \gamma_0 C_{34}}{2M} \quad \xi_1 = \xi_0 + \mu l \\ D = \dfrac{(\xi_0^2 + \xi_1^2)C_{11} + 2\xi_0\xi_1 C_{12} + 2\gamma_0(\xi_0 C_{13} - \xi_1 C_{14}) + \gamma_0^2 C_{33}}{2M} - \dfrac{\xi_0^2}{2\omega^2} - \dfrac{\gamma_0^2}{2\sigma^2} \end{cases} \tag{15b}$$

$$g_S(\theta|F;l) = \frac{1}{\pi}\sqrt{\frac{A_1}{A}}\ \frac{\exp[-D - v(vA+2B)]\left[1-\sqrt{\pi}\ \exp\left[\dfrac{(B+\mu A)}{A}\right]^2 \dfrac{B+\mu A}{\sqrt{A}}\ \text{erfc}\left(\dfrac{B+\mu A}{\sqrt{A}}\right)\right]}{\exp\left(\dfrac{B_1^2}{A_1} - C_1 + \dfrac{\xi_0^2}{2\omega^2} + \dfrac{\gamma_0^2}{2\sigma^2}\right)\left[\text{erf}\left(\dfrac{A_1\xi_1 + B_1}{\sqrt{A_1}}\right) + 1\right]} \tag{16a}$$

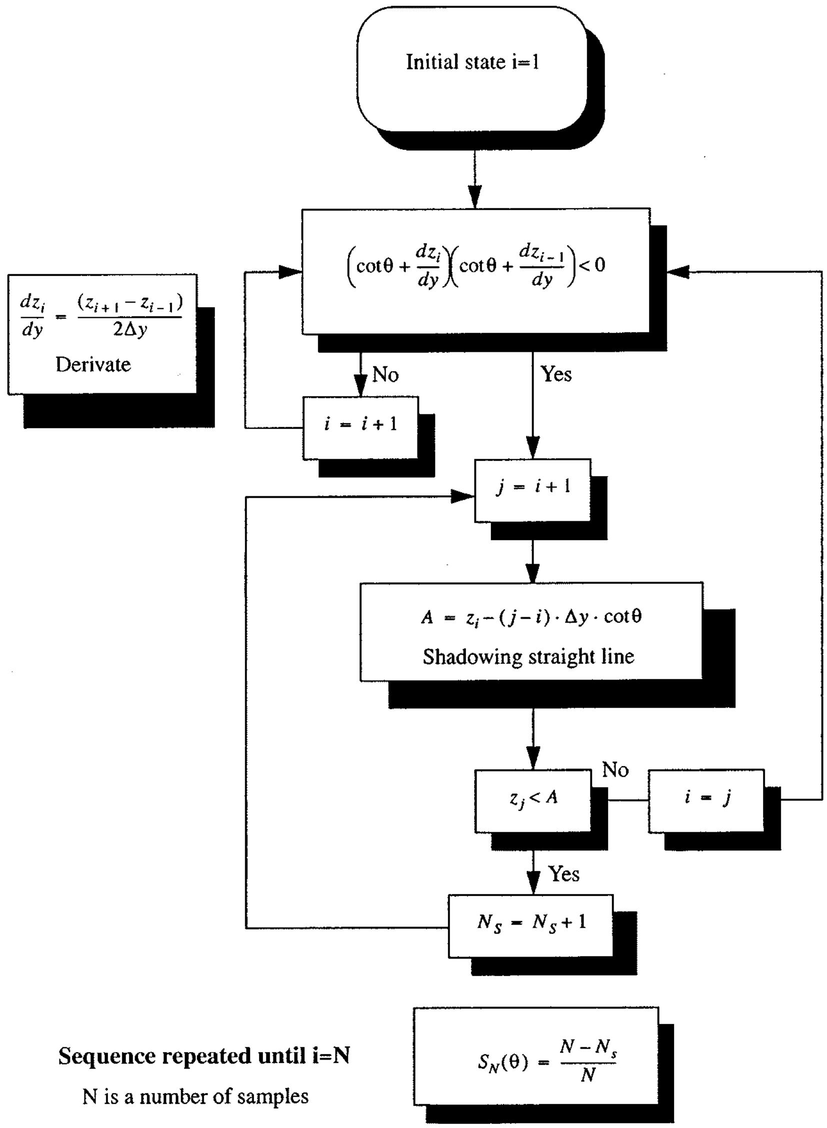

Fig. 4. Algorithm of shadowing function.

$$\cdot\begin{cases} f_M(y) = \dfrac{f_{33}^2(y) - f_{34}^2(y)}{1 - e^{-2y^2}} \\ f_{11}(y) = 1 - e^{-2y^2}(1 - 2y^2 + 4y^4) \\ f_{12}(y) = e^{-3y^2}(1 - 2y^2) - e^{-y^2} \\ f_{33}(y) = 1 - e^{-2y^2}(1 + 2y^2) \\ f_{34}(y) = e^{-y^2}(2y^2 + e^{-2y^2} - 1) \\ f_{13}(y) = 2y^3 e^{-2y^2} \\ f_{14}(y) = ye^{-y^2}(1 - e^{-2y^2}). \end{cases} \quad (19)$$

Using (19), (15b), and (16b) and making the variables transformations

$$\begin{cases} h = \dfrac{\xi_0}{\sqrt{2}\omega} \Rightarrow \dfrac{\xi_1}{\sqrt{2}\omega} + h_1 = h + y\sqrt{2}v \\ \dfrac{\gamma_0}{\sqrt{2}\sigma} = v - p \quad \text{with} \quad v = \dfrac{\mu}{\sqrt{2}\sigma} \end{cases} \quad (20)$$

we obtain (21), shown at the bottom of the next page.

Therefore, the Smith and Wagner conditional probabilities depend on four variables $\{h, p, v, y\}$ and functions $\{f_{ij}(y); f_M(y)\}$. According to (3), the averaged shadowing function, for a Gaussian stationary process with mean zero and for infinite observation length, is given by

$$S(\theta) = \frac{1}{2\pi\sigma\omega}\int_{-\infty}^{\infty}\int_{-\infty}^{\mu}\exp\left(-\frac{\xi_0^2}{2\omega^2} - \frac{\gamma_0^2}{2\sigma^2}\right) \cdot \exp\left(-\int_0^{\infty} g(\theta|F;l)\,dl\right)\,d\xi_0\,d\gamma_0. \quad (22)$$

Using the variables transformation of (20), (22) becomes

$$S(v) = \frac{1}{\pi}\int_{-\infty}^{\infty}\int_0^{\infty}\exp[-h^2 - (v-p)^2] \cdot \exp\left[-L_c\int_0^{\infty} g(y,h,p,v)\,dy\right]\,dh\,dp. \quad (23)$$

The integration interval of function g is defined between zero and the infinite. In order to reduce this domain, the integral is separated into two parts

$$\int_0^{\infty} g\,dy = \int_0^{y_t} g\,dy + \int_{y_t}^{\infty} g\,dy = G + G_t. \tag{24}$$

The transition integration boundary y_t is obtained when

$$f_{ij} = \delta_{ij} = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{for } i \neq j. \end{cases} \tag{25}$$

The terms f_{ij} (19) are represented in Fig. 3 versus the reduced variable y. We observe that the intercorrelation functions $\{f_{12}; f_{34}; f_{13}; f_{14}\}$ are very weak for $y \geq 3$ and that the correlation functions $\{f_{11}; f_{33}\}$ become independent of y and are equal to one, therefore, $y_t = 3$. In fact, (25) corresponds to Wagner and Smith hypothesis, moreover, they assume that $y_t = 0$. Substituting (25) into (21), we obtain

$$\begin{cases} D = h^2 + h_1^2 + (v-p)^2 \\ \dfrac{B+\mu A}{\sqrt{A}} = v \\ \dfrac{\sqrt{f_M}}{\pi\sqrt{2}f_{33}} = \dfrac{1}{\pi\sqrt{2}} \\ \mu(\mu A + 2B) = v^2 \end{cases} \qquad \begin{cases} C_1 = h^2 + (v-p)^2 \\ \dfrac{B_1}{\sqrt{A_1}} = 0 \\ \sqrt{A_1} = 1 \quad \dfrac{L}{\pi}\sqrt{\dfrac{A_1}{A}} = \dfrac{\sqrt{2}}{\pi}. \end{cases} \tag{26}$$

Substituting (26) into (15a), (16a) and into (24), $\{G_{tW}, G_{tS}\}$ are given by

$$\begin{cases} G_{tW}(h,v) = \dfrac{\Lambda}{2}\,\mathrm{erfc}(h + y_t\sqrt{2}v) \\ G_{tS}(h,v) = -\ln\left[1 - \dfrac{\mathrm{erfc}(h + y_t\sqrt{2}v)}{2}\right]^{\Lambda}. \end{cases} \tag{27}$$

Using (24) and (23), the Wagner and Smith shadowing functions are defined as follows:

$$S_{W,S}(v) = \frac{1}{\pi}\int_{-\infty}^{\infty} I(h)\left[\int_0^{\infty} J(h,p)\,dp\right] dh \tag{28a}$$

with

$$\begin{cases} I(h) = \exp[-h^2 - G_{tW,tS}(h,v)] \\ J(h,p) = \exp\left[-(v-p)^2 - L_c\displaystyle\int_0^{y_t} g_{tW,tS}(y,h,p,v)\,dy\right]. \end{cases} \tag{28b}$$

Wagner and Smith assume $y_t = 0$ leading to

$$\begin{cases} S_W(v) = \dfrac{(1-e^{-\Lambda})F(v)}{\Lambda} \\ S_S(v) = \dfrac{F(v)}{\Lambda + 1} \end{cases} \quad \text{with} \quad F(v) = 1 - \frac{1}{2}\,\mathrm{erfc}(v). \tag{29}$$

Table I summarizes the calculations of the averaged one-dimensional (1-D) monostatic shadowing function over the slopes p and over the heights h for a Gaussian autocorrelation function. This determination requires three linked integrations. The first calculates the exact integration of function g on the interval $[0; y_t]$. The second one is made over $J(h,p)$ according to the variable p. Thus, the last result obtained is multiplied by $I(h)$ and integrated over h. The Wagner and Smith classical shadowing functions do not take into account the correlation, i.e., y_t is equal to zero. This involves that the $J(h,p)$ integration over p becomes independent of h, then the two integrations over $\{p, h\}$ of $\{J(p), I(h)\}$ are independent and resolved analytically ((29)). Finally, the shadowing function depends on only one parameter $v = \mu/\sigma\sqrt{2}$, where μ represents the slope of incident beam and σ the slopes root mean square.

IV. Simulations

In order to estimate the introduced hypothesis, the shadowing function is determined numerically by generating the surface [8]. It is the reference solution because it does not assume any hypotheses. The surface $s(i)$ is generated by applying to the input of the filter of impulse response $r(i)$, a Gaussian white noise $b(i)$ of unitary variance with zero mean, which involves that $s(i) = b(i) * r(i)$, where $*$ denotes the convolution product.

$$\begin{cases} D = \dfrac{(h^2+h_1^2)f_{11} + 2hh_1f_{12} + 2\sqrt{2}(v-p)(hf_{13} - h_1f_{14}) + (v-p)^2 f_{33}}{f_M} \\ \dfrac{B+\mu A}{\sqrt{A}} = \dfrac{\sqrt{2}(hf_{14} - h_1f_{13}) + (v-p)f_{34} + vf_{33}}{\sqrt{f_{33}f_M}} \quad \dfrac{L_c\sigma\omega}{4\pi A\sqrt{M}} = \dfrac{\sqrt{f_M}}{\pi\sqrt{2}f_{33}} \\ \mu(\mu A + 2B) = \dfrac{v^2 f_{33} + 2\sqrt{2}v(hf_{14} - h_1f_{13}) + 2vf_{34}(v-p)}{f_M} \\ C_1 = h^2\,\dfrac{f_{11}f_{33} - 2f_{14}^2}{f_{33}f_M} + (v-p)^2\dfrac{f_{33}^2 - f_{34}^2}{f_{33}f_M} + 2h(v-p)\sqrt{2}\,\dfrac{f_{13}f_{33} - f_{14}f_{34}}{f_{33}f_M} \\ \dfrac{B_1}{\sqrt{A_1}} = \dfrac{h(f_{12}f_{33} + 2f_{14}f_{13}) + (v-p)\sqrt{2}(f_{13}f_{34} - f_{14}f_{33})}{\sqrt{f_{33}f_M(f_{11}f_{33} - 2f_{13}^2)}} \\ \sqrt{A_1}\xi_1 = h_1\sqrt{\dfrac{f_{11}f_{33} - 2f_{13}^2}{f_{33}f_M}} \quad \dfrac{L_c}{\pi}\sqrt{\dfrac{A_1}{A}} = \dfrac{\sqrt{2}}{\pi}\,\dfrac{\sqrt{f_{11}f_{33} - 2f_{13}^2}}{f_{33}} \end{cases} \tag{21}$$

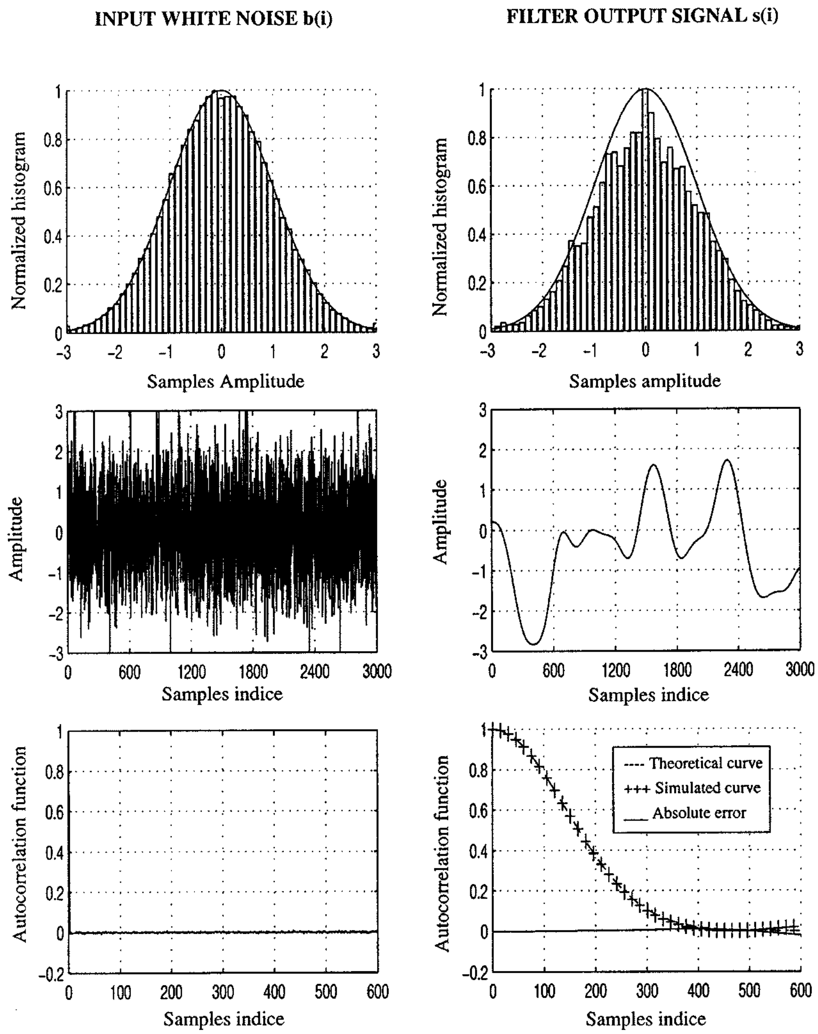

Fig. 5. Random surface generation of Gaussian autocorrelation function.

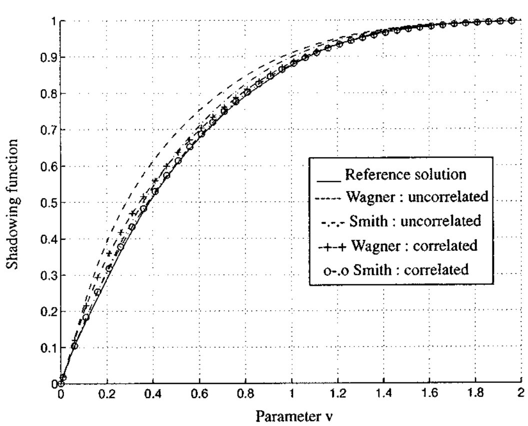

Fig. 6. One-dimensional monostatic shadowing function.

For a Gaussian autocorrelation function of unitary variance, its impulse response is given by

$$r(i) = \sqrt{\frac{2}{L_c\sqrt{\pi}}} \exp\left(-\frac{2i^2}{L_c^2}\right). \tag{30}$$

In order to estimate the hypothesis introduced by Wagner and Smith, the uncorrelated (29) and correlated (Table I) shadowing functions are compared with the reference solution. Fig. 6 depicts the different shadowing functions. The reference solution is in solid line, the Wagner and Smith results without the correlation are plotted in dotted line, whereas the correlation is included in crosses and circles. We observe that the effect of the correlation decreases the values of the shadowing function with an identical behavior. In Fig. 7, the absolute error between the reference solution and the Wagner and Smith shadowing functions are shown. For values of $v \geq 1.4$, the effect of the correlation is negligible, corresponding to incidence angles inferior or equal to $\theta_C = \text{atan}(0.5/\sigma)$, where σ^2 is the slopes variance. For values of $v < 1.4$, the correlation divides the absolute error by about three (Table II). The results obtained by Smith are better than those determined by Wagner.The shadowing function is determined numerically by applying the algorithm of Fig. 4 [8]. The output and input signals of the filter are represented at the top of Fig. 5 for a Gaussian autocorrelation function. The white noise generated is composed of $100\,000 = 500L_c$samples and the correlation length L_c is equal to 200. The first two figures represent the input and the output normalized histograms, com-

TABLE I
WAGNER'S AND SMITH'S SHADOWING FUNCTIONS FOR A GAUSSIAN AUTOCORRELATION FUNCTION

Shadowing function	$S(v) = \frac{1}{\pi}\int_{-\infty}^{\infty} I(h)\left[\int_0^{\infty} J(h,p)dp\right]dh$ $I(h) = \exp[-h^2 - G_t(h,v)]$ $J(h,p) = \exp\left[-(v-p)^2 - L\int_0^{y_t} g(y,h,p,v)dy\right]$
Integration	$h \in [-3;3]$ $p \in [0;3+v]$ $y_t = 3$
WAGNER function $g_W L_c$	$g_W L_c = W_{10}\cdot e^{-S_1}\cdot[1 - e^{S^2}\sqrt{\pi}S\,erfc(S)]$ *with* $\begin{cases} S = \frac{B+\mu A}{\sqrt{A}} \\ S_1 = D + \mu(\mu A + 2B) \end{cases}$ *and* $\begin{cases} S = \frac{\sqrt{2}(hf_{14} - h_1 f_{13}) + (v-p)f_{34} + vf_{33}}{\sqrt{f_{33}f_M}} \quad W_{10} = \frac{\sqrt{f_M}}{\pi\sqrt{2}f_{33}} \\ D = \frac{(h^2+h_1^2)f_{11} + 2hh_1 f_{12} + 2\sqrt{2}(v-p)(hf_{13} - h_1 f_{14}) + (v-p)^2 f_{33}}{f_M} - h^2 - (v-p)^2 \\ \mu(\mu A + 2B) = \frac{v^2 f_{33} + 2\sqrt{2}v(hf_{14} - h_1 f_{13}) + 2vf_{34}(v-p)}{f_M} \end{cases}$
WAGNER function G_{tW}	$G_{tW} = \frac{\Lambda}{2}\cdot erfc(h + y_t\sqrt{2}v)$ $v = \frac{\mu}{\sigma\sqrt{2}}$ $\Lambda = \frac{e^{-v^2} - v\sqrt{\pi}erfc(v)}{2v\sqrt{\pi}}$
SMITH function $g_S L_c$	$g_S L_c = \frac{1}{\pi}\sqrt{\frac{A_1}{A}}\cdot\frac{e^{-S_1}\cdot[1 - e^{S^2}\sqrt{\pi}S\,erfc(S)]}{e^{S_2}\cdot erfc(-S_3)}\cdot e^{-h^2-(v-p)^2}$ *with* $\begin{cases} S_2 = \frac{B_1^2}{A_1} - C_1 \\ S_3 = \frac{A_1 h_1 + B_1}{\sqrt{A_1}} \end{cases}$ *and* $\begin{cases} C_1 = h^2\cdot\frac{f_{11}f_{33} - 2f_{14}^2}{f_{33}f_M} + (v-p)^2\cdot\frac{f_{33}^2 - f_{34}^2}{f_{33}f_M} + 2h(v-p)\sqrt{2}\cdot\frac{f_{13}f_{33} - f_{14}f_{34}}{f_{33}f_M} \\ \frac{B_1}{\sqrt{A_1}} = \frac{h(f_{12}f_{33} + 2f_{14}f_{13}) + (v-p)\sqrt{2}(f_{13}f_{34} - f_{14}f_{33})}{\sqrt{f_{33}f_M(f_{11}f_{33} - 2f_{13}^2)}} \\ \sqrt{A_1} = \sqrt{\frac{f_{11}f_{33} - 2f_{13}^2}{f_{33}f_M}} \quad \frac{1}{\pi}\sqrt{\frac{A_1}{A}} = \frac{\sqrt{2}}{\pi}\frac{\sqrt{f_{11}f_{33} - 2f_{13}^2}}{f_{33}} \end{cases}$
SMITH function G_{tS}	$G_{tS} = -\ln\left[1 - \frac{erfc(h + y_t\sqrt{2}v)}{2}\right]^{\Lambda}$

TABLE II
ROOT MEAN SQUARE ERRORS OF SHADOWING FUNCTIONS

Shadowing functions	Decorrelated Wagner	Deccorelated Smith	Correlated Wagner	Corraleted Smith
RMS en %	3,2	1,3	3,0	0,4

pared with their theoretical distribution. We observe a Gaussian behavior with zero mean (centered on zero), the power is contained between -3σ and 3σ. The behavior remains Gaussian because the filter is linear.

The input and output signals, respectively, are plotted in the middle of Fig. 5. The input signal is very noisy whereas the output signal becomes smoother thanks to the correlation. Finally, at the bottom of Fig. 5 are represented the behaviors of the normalized autocorrelation functions. In the input, we observe a peak centered on zero, which is theoretically the Dirac distribution $\delta(i)$, whereas in the output we observe the expected autocorrelation function because the difference between the theoretical curve and the one determined from filter coefficients is small.

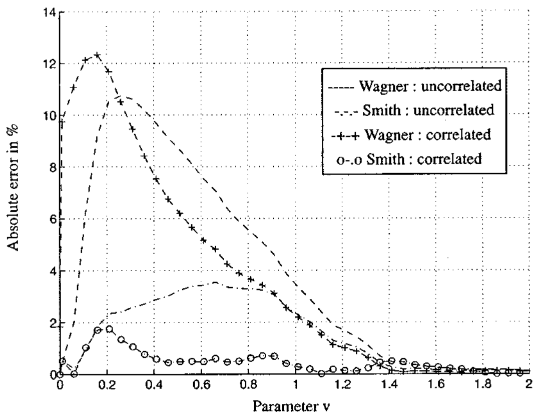

Fig. 7. Absolute error of shadowing function.

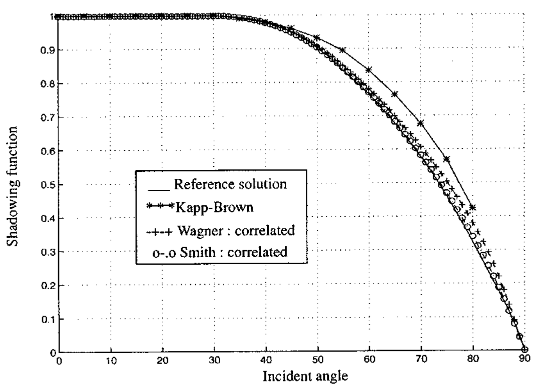

Fig. 8. Comparison of Kapp's and Brown's models to our results for $\sigma^2 = 0.3$.

In [9], Kapp and Brown have calculated the averaged shadowing function over the heights $S(\theta, \gamma_0)$ from the first three terms of Rice's series while including the correlation. In order to compare their results with ours, those are multiplied by $1 - \text{erfc}(v)/2$, representing the slopes integration γ_0. The Wagner and Smith correlated shadowing functions, that of Kapp–Brown, and the reference solution are plotted in Fig. 8 versus the incidence angle for a variance slopes $\sigma^2 = 0,3$. We observe that the Kapp and Brown results are less accurate than ours and their model diverges at grazing angles.

V. Conclusion

In this article, we have quantified the effect of the correlation on the Wagner and Smith 1-D monostatic shadowing functions. Ricciardi and Sato proved that the shadowing function is equal to an infinite series of Rice (4). We showed that the obtained uncorrelated analytical resulthas not physical sense. On the other hand, the Wagner solution given by the first term of this series is correct, but overestimates the shadowing function. In order to estimate the effect of the correlation, the Wagner and Smith formulations are used by considering a correlated stationary Gaussian process, for a Gaussian autocorrelation function. So, the Wagner and Smith correlated results are lower than the uncorrelated ones but are greater than those obtained by the reference solution. Nevertheless, the Smith model is very close to the reference solution (Figs. 6, 7) and we show that the effect of the correlation is negligible when the incidence angle is inferior to atan$(0.5/\sigma)$, where σ^2 is the slopes variance. Finally, the analytical expressions of the shadowing functions given in this article allow to obtain a best accurate than those presented in [9]. The prospect of this work is to extend the method in the two-dimensional configuration, i.e., introduce the second component x. In this case, the spatial autocorrelation function $R_0(x, y)$ depends on the Cartesian coordinates (x, y), which involves that we must give the new expression of the covariance matrix $[C]$ as a function of $R_0(x, y)$. In literature, we have found any paper, which presents this aspect.

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